The Math behind the Music
Leon Harkleroad
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review by
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Mathematics and Music: there are several interesting connections, and a number of books which deal with various aspects of this interplay have been published during the last years. Whereas [1] and [2] are collections of articles written by different people the book under review is the author's summary of a number of courses he has given at Cornell University.

As a natural starting point the problem of how to construct scales is chosen. How is the Pythogerean scale definied, what are just intonations, what is the equal temperament and how comes the twelfth root of two into play? Harkleroad uses the opportunity to explain also some fundamental facts on Fourier analysis.

In the next chapter some elementary procedures how to vary a musical theme are explained in the language of group theory. „T_7", the transposition of 7 semitones, has T_{-7} as its inverse etc. Here one has to calculate modulo 12, and the reader can learn how arithmetic modulo a fixed integer is defined. As illustrations of these theoretical facts many examples are included. E.g., in *Pomp and Circumstance* (by E. Elgar) the main theme is varied by using transpositions whereas inversion plays a crucial role in *The muscal offering* (by J.S. Bach). It is very helpful that a compact disc is included which allows one to compare the theoretical facts with the real sound.

Then the „art of bell ringing" is described. The interest in this aspect of „mathematics and music" is rather recent. The presentation here is more elementary than that in [2] (the article of Roaf and White) and [3], and the emphasis is on the connections with group theory. „Subgroups" and „cosets" enter the stage, and a modestly complicated example of a bell-tower rendition can be found on the CD.

The next chapter is devoted to „Music by chance" (an aspect which, surprisingly, is not covered by the other books on this subject). The idea came up in the eighteenth century, and many composers – among them Haydn and Mozart – have presented such compositions. The idea ist simple. First write a finite number of bars which can serve as the first bar of a menuet, say. Then continue with several suggestions for bar two, and so on. Randomness comes into play when producing a concrete menuet from these suggestions. For example, if there are 11 suggestions for bar number one which are numbered 2, 3, . . . , 12, throw two dice and select that bar which corresponds to the total sum. Find bar number two in a similar way and continue, in the end you will have a complete „random menuet". The number of different pieces is gigantic, and, whereas masterpieces are not to be expected, it is interesting to hear that some of them sound rather interesting.

(Admittedly, after listening to the first few examples one has the impression to know them all.)

This, however, is not the end of the story. It is also explained how more elaborated stochastical procedures like Markov chains are used to produce musical pieces. The reader can learn here some basic notions from probability theory, e.g. conditional probability.

The step from random selection to deterministic procedures which use geometrical forms or are seemingly stochastic is not very large. The author reports on compositions which convert the shape of the mountains close to Rio de Janeiro into music or which use Lindenmayer systems (these are recursively defined, after some steps one arrives at structures which are self-similar and „chaotic“).

The title of the final chapter is „How *not* to mix mathematics and music“. One learns that numerical relations in musical pieces have been over-emphasized occasionally. For example, it is not hard to find the Fibonacci numbers or (more or less convincing approximations of) the golden ratio in any given composition if one juggles with the number of bars, notes, themes etc.

The book can be recommended as a first survey. Those who want to know more may find many suggestions for further study in the extensive bibliography. It adresses to readers without any mathematical knowledge. Everything is explained very clearly and at great length, and the theoretical facts are illustrated by many pictures, scores and the tracks of the accompanying CD.

It should be noted, however, that also this book cannot give a substantial answer to the question *why* music is such a fascinating part of our life. Mathematics only helps to explain some rather superficial aspects of music or to understand how certain contemporary composers use mathematical methods. But probably it would be rather naive to expect more.

[1] G. Assayag, H.G. Feichtinger, J.F. Rodriguez (ed.): *Mathematics and Music*. Springer Verlag, New York, 2002.

[2] J. Fauvel, R. Flood, R. Wilson (ed.): *Music and Mathematics*. Oxford University Press, Oxford, 2003.

[3] B. Polster: The mathematics of juggling. Springer Verlag, New York,2003.